Inter-School Mathematical Olympiad

ISMO 2024

Time allowed: 2 hours. There are 10 problems, each worth 7 marks. Questions are roughly in difficulty order though not necessarily so. *Good luck!*

Problem 1. *Find all primes that leave prime remainders when divided by all other primes.*

Problem 2. *In regular heptagon $ABCDEFG$, AB and CE intersect at P. Find the value of $\angle PDG$.*

Problem 3. *Let a and b be real numbers satisfying*

$$(2025a - 2024b)^2 = 2025a^2 - 2024b^2$$

Prove that $a^3 + b^2 = b^3 + a^2$.

Problem 4. *Integers $1, 2, 3, \ldots, n$ are written on a whiteboard where $n > 2$. Each minute, 2 random integers a, b on the board are chosen and replaced with $2a-b$ and $2b-a$. Let $f(n)$ denote the maximum number of odd integers on the board. Find $f(n)$.*

Problem 5. *Let I, S, M, O be positive integers such that*

$$I \times S + M \times O = 2023$$

$$I + S \times M \times O = 2024$$

Find all possible values of $I \times S \times M \times O$.

Problem 6. *Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that*

$$f(x - y) = f(x) - 2xy + y^2$$

Problem 7. *How many ways can a coin be flipped n times and not land on heads or tails 3 times in a row?*

Problem 8. *The number 1 is written on a blackboard. James and Brian play a game, on their turn, they pick a divisor n of $2024!$ and replace the number x on the blackboard by $n + x$ or nx. James goes first. Each divisor may only be picked once. If the number on the board is even at the end of the game, James wins. Otherwise, Brian wins. Determine whether James or Brian has a winning strategy.*

Problem 9. *Convex quadrilateral $ABCD$ satisfies $AB = 9, BC = 11, CD = 6, DA = 6$, and BD bisects $\angle ABC$. Find $AC \times BD$.*

Problem 10. *Let $x_1, x_2, x_3, \ldots$ be an infinite sequence of positive integers. For any x_i with prime factorization $x_i = {p_1}^{\alpha_1} {p_2}^{\alpha_2} \ldots {p_k}^{\alpha_k}$, define $x_{i+1} = (p_1 + 3)^{\alpha_1} (p_2 + 3)^{\alpha_2} \ldots (p_k + 3)^{\alpha_k}$.*

Show that for all starting values of $x_1 \geq 2$, there exists a positive integer N such that $x_{i+2} \geq {x_i}^3$ for all $i \geq N$.

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1. *Do NOT turn over the exam paper until told to do so.*
2. *You will have 2 hours to complete the examination.*
3. *Discussion about the exam is prohibited until 11:59pm Monday 8th April. Anyone found discussing the problems before then is automatically disqualified from the competition.*
4. *Each question is worth 7 marks.*
5. *Solutions with explanations are required. A bare answer is usually worth between 0 and 2 marks only.*
6. *One or two full solutions are likely to earn much more marks than attempts on all 10 problems.*
7. *No electronic devices other than 1 approved non-algebraic function calculator is allowed, graphical instruments (but not protractors) are permitted.*
8. *Start ALL new questions on a new page.*
9. *All questions about the exam paper must be made within the first 30 minutes. If you have a question, you must write on a separate sheet of A4 paper with the problem number and your question and raise your hand. Wait patiently until an examiner comes to you and collect the sheet of paper. You must not talk during the process.*
10. *If you wish to go toilet during the examination, raise your hand and request to go to toilet via the same format as asking a question. You must not talk during the process.*
11. *Write on one side of the A4 paper only and clearly label each page with your name, year level, question number, and page number. If you write on both, one of the sides will not be marked.*
12. *Any violation will invoke a penalty ranging from mark deduction to disqualification.*
13. *You may leave early, but you must not leave in the last 20 minutes of the exam. If you insist to do so, your score may be invalidated.*