Inter-School Mathematical Olympiad

ISMO Summary 2025

1 Results

1.1 Top Scorers

Senior:
Tied First Place: Johan Chan (King's College), Kyle Tac (Rangitoto College)
Third Place: Devin Xu (Pinehurst)

Junior:
First Place: Eric Xiao (Rangitoto College)
Second Place: Remi Geron (St Kentigerns college)
Third Place: Taichu Hao (Auckland Grammar)

1.2 Medal List

Devin	Xu	Pinehurst	Senior	GOLD
Elijah	Tac	Rangitoto	Senior	GOLD
Enzo	Li	Long bay	Senior	GOLD
Eric	Xiao	Rangitoto	Junior	GOLD
Isabelle	Ning	Kristin	Junior	GOLD
Isla	Wang	Macleans	Junior	GOLD
Jackie	Xu	St Cuthberts	Junior	GOLD
Jennifer Yuhan	Chai	Baradene	Senior	GOLD
Jerry	Li	Pinehurst	Senior	GOLD
Johan	Chan	Kings	Senior	GOLD
Keisuke	Ohta	Rangitoto	Senior	GOLD
Kyle	Tac	Rangitoto	Senior	GOLD
Leo	Wang	Rangitoto	Senior	GOLD
Lifeng	Song	Grammar	Junior	GOLD
Logan	Murphie	Pinehurst	Junior	GOLD
Remi	Geron	St Kents	Junior	GOLD
Taichu	Hao	Grammar	Junior	GOLD
Tymon	Mieszkowski	Long bay	Senior	GOLD
WenYao	Zhong	Pinehurst	Junior	GOLD
Yixiang	Xu	Macleans	Senior	GOLD
Benjamin	Chan	Macleans	Junior	SILVER

Bosco	Jin	Pinehurst	Junior	SILVER
Chanwoo	Eu	Kings	Senior	SILVER
Connor	Hu-Wen	Macleans	Junior	SILVER
Dingding	Mao	Rangitoto	Junior	SILVER
Emily	Chen	Rangitoto	Senior	SILVER
Eric	Shen	Macleans	Junior	SILVER
Ethan	Wang	Rangitoto	Junior	SILVER
Felix	Luo	Rangitoto	Junior	SILVER
Gordon	Peng	Rangitoto	Junior	SILVER
Haoting	Lu	Rangitoto	Junior	SILVER
Helen	Chen	Rangitoto	Senior	SILVER
Jack	Chen	Kings	Junior	SILVER
Jack	Chen	Pinehurst	Senior	SILVER
James	Barrington	Kings	Senior	SILVER
Jaylen	Ling	Rangitoto	Senior	SILVER
Jennifer	Lan	Rangitoto	Junior	SILVER
Jo-Ann	Cai	Rangitoto	Junior	SILVER
Jonathan	Xu	Rangitoto	Junior	SILVER
Josephine	Li	ACG	Senior	SILVER
Justin	Lu	ACG	Junior	SILVER
Liam Irwin	O'Grady	ACG	Senior	SILVER
Osbert	Gu	Pinehurst	Senior	SILVER
Richie	Liu	Kristin	Senior	SILVER
Rick	Tang	Rangitoto	Senior	SILVER
Selina	Ni	Rangitoto	Senior	SILVER
Sivaram	Kolla	ACG	Junior	SILVER
Teddy	Tang	ACG	Junior	SILVER
Theodore	Zhang	ACG	Junior	SILVER
Wanyu	Liang	Pinehurst	Senior	SILVER
William	Huang	Kristin	Senior	SILVER
Wilson	Lin	Kristin	Junior	SILVER
Zhiyuan	Yu	Macleans	Junior	SILVER
Zhonglin	Yu	Macleans	Junior	SILVER
Ada	Zhang	Kristin	Senior	BRONZE
Alan	Li	Rangitoto	Junior	BRONZE
Alina	Chen	St Cuthberts	Junior	BRONZE
Andrew	Li	St Kents	Junior	BRONZE
Andrew	Xue	Rangitoto	Junior	BRONZE
Betty	Liu	St Cuthberts	Junior	BRONZE
Bill	Jinheng	ACG	Senior	BRONZE
Branden	Li	Grammar	Junior	BRONZE
Cody	Yang	Rangitoto	Junior	BRONZE
Derek	Shi	Kings	Junior	BRONZE

Doris	Xiao	Pinehurst	Junior	BRONZE
Eason	Guo	Kristin	Junior	BRONZE
Ellie	Siu	St Cuthberts	Senior	BRONZE
Enpei	Lin	Grammar	Junior	BRONZE
Eric	Ni	Rangitoto	Junior	BRONZE
Ethan	Sheng	Kings	Junior	BRONZE
Euan	Craig	ACG	Senior	BRONZE
Gavin	Jang	Rangitoto	Senior	BRONZE
Gavin	Xin	ACG	Junior	BRONZE
Giavou	Xu	Kings	Senior	BRONZE
Haoen	Samuel	ACG	Junior	BRONZE
Harold	MacCulloch	Grammar	Junior	BRONZE
Henry	Wang	Westlake	Senior	BRONZE
Hugo	Tan	Rangitoto	Senior	BRONZE
Ian	Chen	Kings	Junior	BRONZE
Jack (Ying Yuan)	Zhang	Rangitoto	Junior	BRONZE
James	Park	Westlake	Junior	BRONZE
Jason	Li	Macleans	Senior	BRONZE
Jerry	Ma	ACG	Junior	BRONZE
Jerry	Yang	Pinehurst	Junior	BRONZE
Jessie	Dong	Rangitoto	Junior	BRONZE
Jiming	Yang	Rangitoto	Junior	BRONZE
Joseph	Xin	Grammar	Senior	BRONZE
Julian	Yoon	Rangitoto	Senior	BRONZE
Kai	Li	Kings	Junior	BRONZE
Karol	Zhang	St Kents	Y7-8	BRONZE
Karry	Gong	Long bay	Junior	BRONZE
Ken	Li	Rangitoto	Senior	BRONZE
Linghan	Meng	Pinehurst	Senior	BRONZE
Minu	Park	Grammar	Senior	BRONZE
Oscar	Li	Rangitoto	Junior	BRONZE
Owen	Lin	Westlake	Junior	BRONZE
Patrick	Nguyen	Rangitoto	Junior	BRONZE
Phoenix	McClean	Rangitoto	Senior	BRONZE
Sam	Qing	Kristin	Junior	BRONZE
Selena	Chen	Long bay	Junior	BRONZE
Tony	Guo	Kristin	Junior	BRONZE
Wang Xuan	Qiu	Kristin	Junior	BRONZE
YiMing	Luo	ACG	Unknown	BRONZE
Yiqing	Liu	Rangitoto	Junior	BRONZE
Youran(Yolanda)	Zhang	Rangitoto	Senior	BRONZE
Yuanyuan	Zheng	Rangitoto	Senior	BRONZE
Zihao	Zhou	Rangitoto	Senior	BRONZE

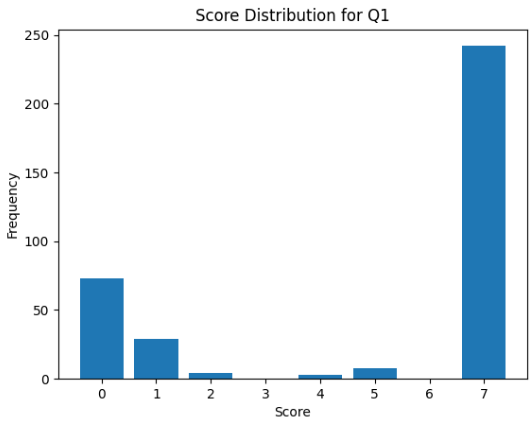
Zitian (Joshua)	Mu	ACG	Senior	BRONZE

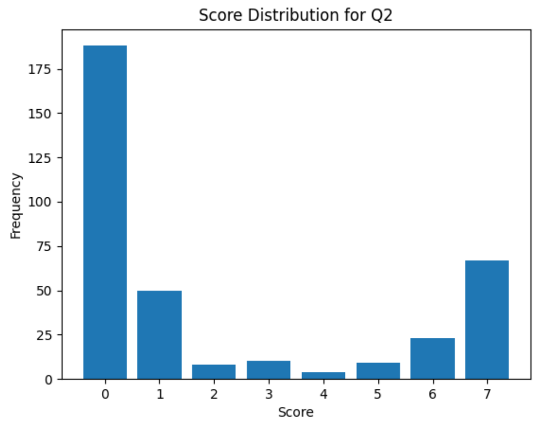
1.3 Award Details

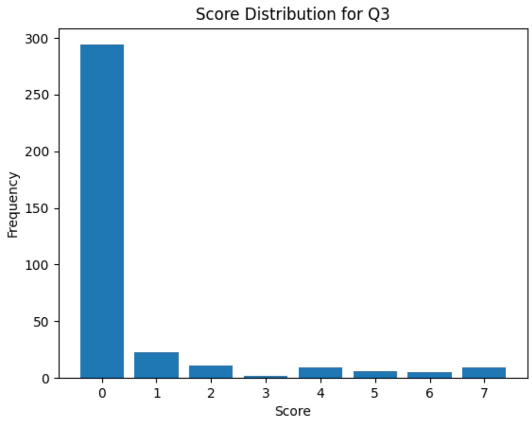
Gold: 19 Students, cut off = 29 points
Silver: 35 Students, cut off = 21 points
Bronze: 54 Students, cut off = 14 points

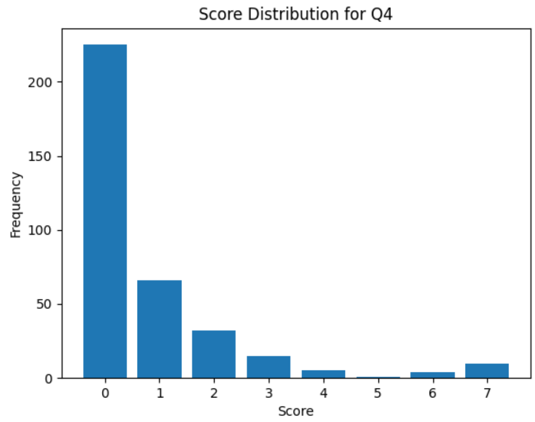
2 Score Distribution

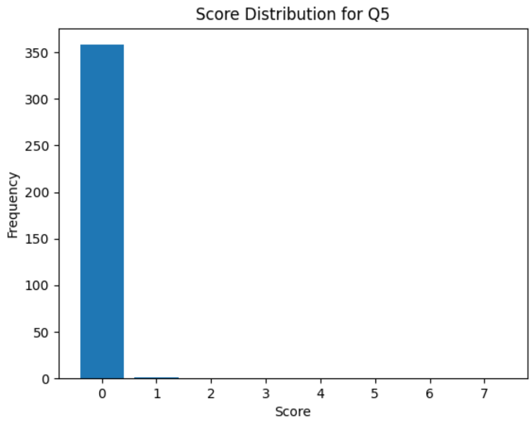
2.1 Score Distribution by Question

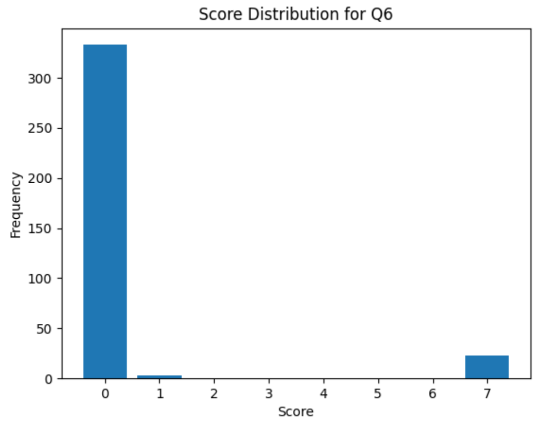


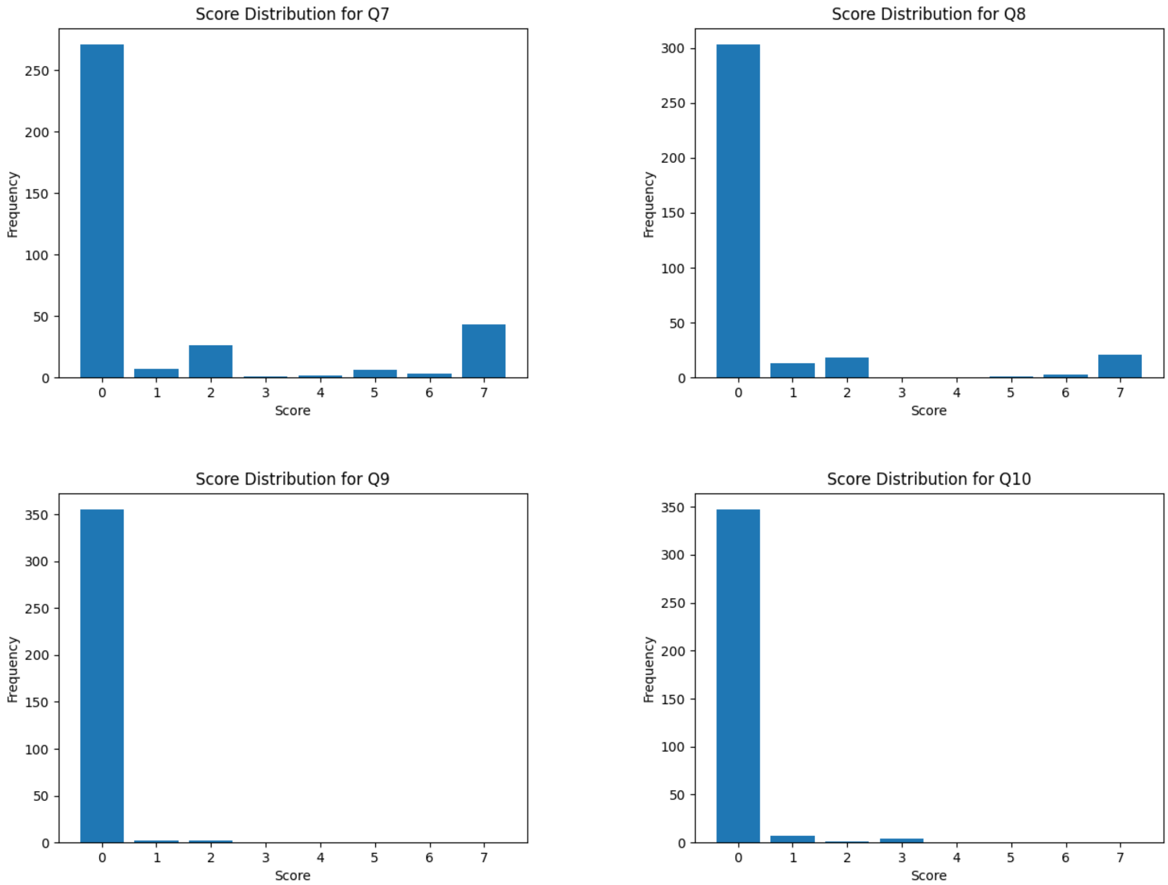











Score Distribution for Q7
Frequency
Score
0
50
100
150
200
250
0 1 2 3 4 5 6 7
Score Distribution for Q8
Frequency
Score
0
50
100
150
200
250
300
0 1 2 3 4 5 6 7
Score Distribution for Q9
Frequency
Score
0
50
100
150
200
250
300
350
0 1 2 3 4 5 6 7
Score Distribution for Q10
Frequency
Score
0
50
100
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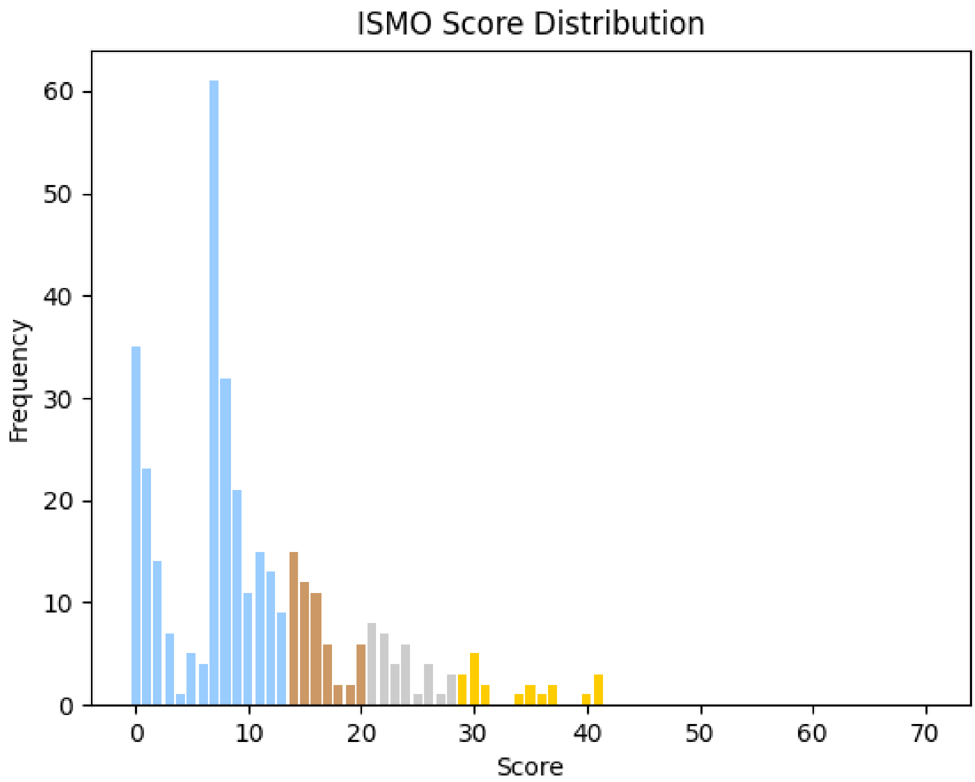
200
250
300
350
0 1 2 3 4 5 6 7

2.2 Overall Score Distribution



2.3 Statistical Data

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Total
Mean	4.97	2.13	0.59	0.82	0.00	0.46	1.17	0.61	0.02	0.06	10.81
Median	7	0	0	0	0	0	0	0	0	0	8.00
Mode	7	0	0	0	0	0	0	0	0	0	7.00

3 Solutions

Problem 1. *Calculate the numerical value of the expression:*

$$\binom{8}{4} + \binom{9}{4} + \binom{10}{4} + \binom{11}{4} + \binom{12}{4} + \binom{13}{4} + \binom{14}{4} + \binom{15}{4}$$

Where $\binom{m}{n} = \frac{m!}{n!(m-n)!}$, *and* $n! = n \times (n-1) \times \ldots \times 1$

Solution (by James Xu): *We'll state a well known lemma:*

$$\binom{m}{n} + \binom{m}{n+1} = \binom{m+1}{n+1}$$

Proof (not required by students):

$$\begin{aligned} LHS &= \frac{m!}{n!(m-n)!} + \frac{m!}{(n+1)!(m-n-1)!} \\ &= \frac{m!}{n!(m-n-1)!}\left(\frac{1}{m-n} + \frac{1}{n+1}\right) \\ &= \frac{m!}{n!(m-n-1)!}\frac{m+1}{(m-n)(n+1)} \\ &= \frac{(m+1)!}{(m-n)!(n+1)!} = RHS \end{aligned}$$

Using this, we can see that the equation is

$$\binom{8}{5} + \binom{8}{4} + \binom{9}{4} + \binom{10}{4} + \binom{11}{4} + \binom{12}{4} + \binom{13}{4} + \binom{14}{4} + \binom{15}{4} - \binom{8}{5}$$
$$= \binom{16}{5} - \binom{8}{5}$$
$$= 4312$$

Marker's Report

1. *This problem was by far the most solved problem in the paper.,*

2. *Most contestants either got 7/7 or 0/7 for this problem. Some contestants made minor arithmetic errors which resulted in some point deductions, and some others got a mark for evaluating a single binomial coefficient (for example,* $\binom{8}{4} = 70$*).,*

3. *The contestants who got 0 were mostly unable to understand the notation of binomial coefficients. The most common mistake was treating the terms as fractions (e.g.* $\binom{10}{4}$ *becoming* $\frac{10}{4} = 2.5$*).*

Problem 2. *Find all integers* (a, b) *such that* $6ab + 2a + 7 = 17 - 12b$

Solution (by Eric Liang): *Note that the original equation can be factorized into the following form*

$$(2a + 4)(1 + 3b) = 14$$

By $2a + 4$ *is even, we must have that*

$$2a + 4 = \pm 2, \pm 14$$

So the pairs are $(-1, 2), (5, 0)$*, as the other two pairs have non-integer solutions for* b

Marker's Report

This problem is less scored upon than originally expected. Many students were unable to identify that the expression is factorisable. Many students also lost significant marks for not considering the negative case.

Problem 3. *Let G be the centroid of $\triangle ABC$. Let D, E, F be the centroids of $\triangle ABG, \triangle BCG, \triangle ACG$ respectively.*

Prove that G is the centroid of $\triangle DEF$

Solution (by James Xu): *We'll use the letters to denote the coordinates of the point. Hence, by the center of mass property of centroid:*

$$D = \frac{A+B+G}{3}$$

$$E = \frac{C+B+G}{3}$$

$$F = \frac{A+C+G}{3}$$

And note that

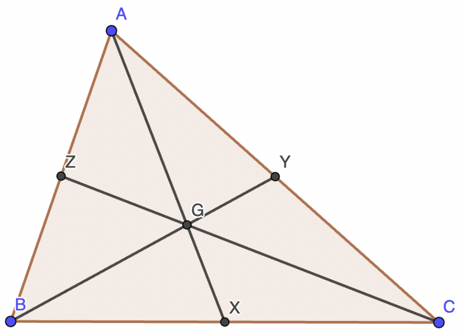
$$G = \frac{A+B+C}{3} = \frac{D+E+F}{3}$$

Hence G is centroid of $\triangle DEF$

Alternate Solution:

Let X, Y, Z be the midpoints of AC, BC, AB respectively, use $A(\triangle RST)$ to denote the area of triangle RST

Lemma (Not required by students to proof): $AG = 2GX, BG = 2GY, CG = 2GZ$



Proof: Since X, Y, Z are midpoints, the following pairs of triangle has same base and height hence equal area:

$$A(\triangle BGX) = A(\triangle CGX) \quad (1)$$
$$A(\triangle CGY) = A(\triangle AGY) \quad (2)$$
$$A(\triangle AGZ) = A(\triangle BGZ) \quad (3)$$
$$A(\triangle BCZ) = A(\triangle ACZ) \quad (4)$$

$$A(\triangle ABY) = A(\triangle BCY) \tag{5}$$

From $(4) - (3)$,

$$A(\triangle AGC) = A(\triangle BGC)$$

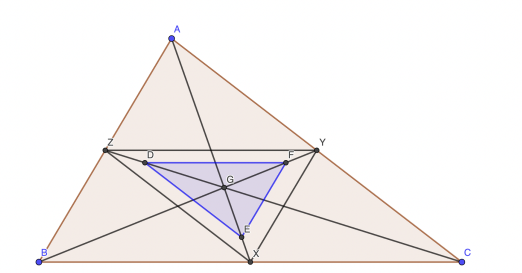
From $(5) - (2)$,

$$A(\triangle AGB) = A(\triangle BGC)$$

Hence, all 6 small triangles has the same area. Therefore

$$2 : 1 = A(\triangle AGB) : A(\triangle BGX) = AG : GX$$

By symmetry, the other two ratios must be $2 : 1$ *too.*



Now, since XY, YZ, ZX *are midlines, they are parallel to the respective sides and* $\triangle XYZ$ $\triangle ABC$ *with centroid* G
Apply lemma in ABG, BCG, CAG *gives*

$$GE = 2EX, GF = 2FY, GD = 2DZ$$

and hence $\triangle EFD$ *is the result of scaling* $\triangle XYZ$ *at* G *with a factor of* $\frac{2}{3}$ *thus must retain the centroid* G

Marker's Report
This is a tough question for most of the people as participants don't have a lot of knowledge on geometry problems. Participants do not understand how to prove centroid identity. Only a little bit amount of the people knows the rule for centroid. And only about 5 participants can provide an expected solution.

Some people make the assumption to say that when the large triangle is regular the statement is true. However that doesn't prove that the statement is true when triangle is not regular triangle.

Some people use coordinate geometry to prove the question, it is doable but probably not the way we expected, so I give some marks depending on the accuracy of their calculation and the precision of their explanation.

Problem 4. *Solve for x:*

$$||\dots|||x-1|-2|-3|\dots|-2024|=2025$$

Solution By Jay Zhao.

For this proof we will define $P_n(x) = ||\dots|||x-1|-2|-3|\dots|-n||$ for all positive integers $n \geq 1$. The problem is equivalent to finding all solutions of the equation $P_{2023}(x) = 2024$.

First note that $P_n(x)$ is non negative for all values of x.

Next, notice that for $n > 1$ we have that if $P_n(x) = m$ where $m > n$ then it must be that $|P_{n-1}(x) - n| = m$. This is true either when $P_{n-1}(x) = m+n$ or $n-m < 0$. But we have already established that $P_n(x)$ is non negative. So it follows that $P_{n-1}(x) = m+n$, but notice that this is also totally greater than $n-1$. So $P_n(x) = m > n$ actually implies $P_{n-1}(x) > m+n > n-1$.

So by induction we have that if $P_{2023} = 2024$ then $|x-1| = P(1) = 2+\dots+n+(n+1) = \frac{n(n+3)}{2}$. This gives us two solutions, either $x = 1 + \frac{n(n+3)}{2}$ or $x = 1 - \frac{n(n+3)}{2}$.

Marker's Report

This problem was attempted by a significant amount of students, and many students received marks on the problem. However, many students scored on the lower end of the problem either due to only having an elementary idea, or calculation errors. Those who managed to solve the problem in full generally received full credits.

Problem 5. *Find the smallest multiple of 2025 that has more than 2025 factors.*

Solution (by James Xu): *We claim the number is* $2^5 \times 3^4 \times 5^2 \times 7^2 \times 11 \times 13 \times 17$

First, we'll state the formula for the number of factors

$$d(n) = (\alpha_1 + 1)(\alpha_2 + 1) \dots (\alpha_k + 1)$$

where $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k}$

Let the number be n*. If* p *is a factor of* n *then all smaller primes are factors of* n*, as if not, then replacing* p *by* q *in the prime factorization of* n *reduces* n*.*

Now consider $n = 2^5 \times 3^4 \times 5^2 \times 7^2 \times 11 \times 13 \times 17$*. If 19 is included, then by lemma the number is a multiple of* $m = 2 \times 3^4 \times 5^2 \times 7 \times 11 \times 13 \times 17 \times 19$*, but* $n < 6m$ *and if we multiply* m *by* $1, 2, 3, 4, 5, 6$ *we get at most 1920 factors.*

If 17 is excluded, we consider $n, 2n, \dots, 16n$*. None of those provide sufficient factors (In practice, very few numbers actually need to be checked since if* $a|b$ *checking* b *automatically checks* a*, and increasing smaller powers are strictly better)*[1]

Marker's Report

Very few students actually attempted this problem with depth. The essence of this problem is simple if the student can grasp the idea that introducing new primes are often more "cost effective" in increasing the number of factors. Many students also expected that problems always have a direct, non-case work solution which is not the case for this problem as students may take multiple guesses to arrive at this number.

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[1]In reality, only 16,15,13,9 needs to be checked

Problem 6. *There are 25 ants on a very thin candy length 1 meter, each ant moves at 1 cm per second towards the end of candy it's facing. When two ants meet, both of them turns around. If an ant reaches the end of the candy it falls off. Show that no ants will be on the candy after 2 minutes.*

Solution (by Haotian Wang): *If we ignore keeping track of which ant is which, two ants colliding and turning around is indistinguishable to the two ants just passing through each other. Therefore, if we swap the identities of the two ants when they collide, each ant will travel in the same direction as when it started until it falls off the candy. Therefore, each ant travels at most 1 meter, which takes at most 100 seconds, which is less than 2 minutes.*

Marker's Report

This is one of the better done problems in the latter half of the exam. A lot of contestants did not read the problem statement properly and assumed the ants are all evenly spread out across the candy. There were many solutions who assumed there exists a "worst" case in which the problem can be discussed, but many failed to justify their claims. Otherwise, most solutions gained a 7, although some lost a mark or two due to clarity in explaining why a collision between ants is the same as them passing through each other.

Problem 7. *Find all integer solutions to the equation* $x^2 - y^2 + x - 5y = 2023$

Solution *By Jerry Dai.*

Note that we can complete the square to get $(x + \frac{1}{2})^2 - (y + \frac{5}{2})^2 = 2017$.

Factoring, we get $(x + y - 3)(x - y - 2) = 2017$.

It is well known that 2017 is prime. Thus, its only factor pairs are (1, 2017), (2017, 1), (-1, -2017), (-2017, -1).

Assume that an integer solution (x, y) exists. Note that x+y-3 is equal to x+y-2y-3=x-y-3 in parity. Then x-y-2 and x-y-3 have different parities, so x+y-3 and x-y-2 must have different parities. But all the factor pairs have equal parity, contradiction. So no integer solutions exist.

Alternative Solution *We can take the equation mod 2 to get that* $x^2 - y^2 + x - y \equiv 1 \mod 2$.

Note that this is equal to $(x^2 + x) - (y^2 + y) \equiv 1 \mod 2$. *Note that* $x^2 + x = x(x + 1) \mod 2$ *must be* 0 *since either* x *or* $x + 1$ *is even, making their product* 0. *Thus the LHS of our equation must be* 0 *while our RHS must be* 1, *contradiction. So no integer solutions exist.*

Aside: *Note that both solutions rely on examining factors of a product to determine that the overall product must be even.*

Marker's Report

This question was solved quite often for its position in the paper. In the future, contestants should try all problems to see if there are any they can make progress on. It was quite easy to get partials for this problem: all the contestant had to do was to complete the square on both x *and* y *to get a difference of squares. Often contestants made the mistake of only using* $x^2 - y^2$ *in their difference of squares factorisation, leading them to be unable to deal with the linear terms left behind.*

After completing the square, many contestants were successfully able to factorise the difference of perfect squares equation, then use the fact that 2017 was prime to provide a parity argument that no integer solutions were possible. Note that contestants who did not know 2017 was prime (it really should have been provided in the paper to be honest) could have found that it was prime in $O(\sqrt(n))$ *time by dividing 2017 by all primes up to the square root of 2017.*

Some contestants used modular arithmetic to provide an argument similar to the parity argument with much less case-checking. Noticing that mod 2 is equivalent to parity simplifies the problem.

Problem 8. *Easter rabbit plans to give out easter eggs to children in 10^{100} different schools, each with 100 students. For each school, the students line up in a line and the rabbit start by offering 1 chocolate to the first student. Each student have an equal chance of taking the egg(s) or saying "double it and give it to the next person" except for the last student, which always take the easter eggs. The rabbit follows the instruction of the students. The process end when any student receives easter eggs and the Easter rabbit moves onto the next school. How many easter eggs should the rabbit expect to give away?*

Solution (by Eric Lee): *Since each student has half chance of taking the chocolate, and half chance of doubling it and giving to the next student, the chance of the n-thstudent being asked by the rabbit at all is $\frac{1}{2^{n-1}}$. e.g. The chance of the first student being asked is 1, second student being asked is 0.5, 100th student $\frac{1}{2^{99}}$ and so on.*

And the chance of that student taking the chocolate is half of that, which is $\frac{1}{2^n}$, with the exception of the 100th student, who has to take the chocolate when asked. e.g. 1st student has 0.5 chance of taking the chocolate, 2nd student 0.25, 99th student $\frac{1}{2^{99}}$, 100th student $\frac{1}{2^{99}}$.

The amount of chocolate a student can receive comes from all the doubling from the previous students. For the nth student, there would have been $n-1$ times the starting 1 chocolate has been doubled. So the nth student gets 2^{n-1} amount of chocolate, if they were to be asked and take the chocolate.

For the first 99 students, they each have a $\frac{1}{2^n}$ chance of taking 2^{n-1} chocolate, making them on average take 0.5 chocolate, or expected value of 0.5. The 100th student has a $\frac{1}{2^{99}}$ chance of getting 2^{99} chocolate, making them on average take 1 chocolate instead. Summing up every student's average gives 50.5. So for 10^{100} schools, the rabbit expects to give out 50.5×10^{100} chocolate

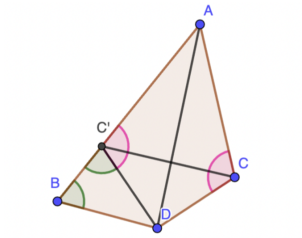
Marker's Report

In general, this problem was attempted and completed more than other past problems in the 7-10 range. However, the majority of the solutions were fakesolves with no real progress made, as contestants simply played around with probabilities. Out of those who came close to the solution, many wrote the expected value of student 100 wrongly, resulting in usually only 2 marks. Also, it is a good idea to write down whatever you have in mind (or hand in your rough works). For example, simply noting that the first student taking 1 chocolate occurs with 50% chance will score you a partial.

Problem 9. *$\triangle ABC$ satisfies $AB = AC$. Let D be a point on segment AC such that D does not coincide with A or C. Let E be the incenter of $\triangle ABD$, and BD intersects AE at F. Prove that the circumcenter of $\triangle BEC$ lies on the the intersection of the circumcircles of $\triangle ABD$ and $\triangle DFC$ other than D.*

Solution (by George Zhu):*Let P' be the circumcenter of $\triangle EBC$. Also let P be the intersection of the circumcircles of $\triangle ABD$ and $\triangle DFC$. We will prove that P' coincides with P.*

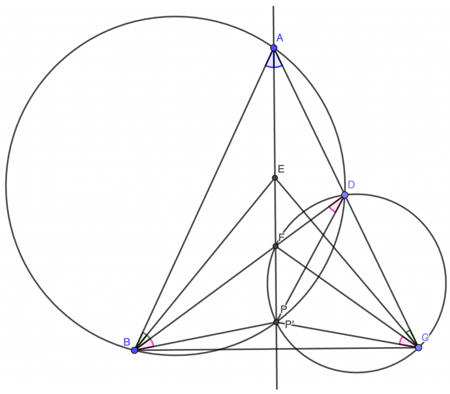
Lemma: *In a convex quadrilateral $ABDC$, if $\angle BAD = \angle CAD$, and $BD = CD$, then A, B, C, D cyclic.*



Proof: WLOG suppose $AB > AC$. Also let the reflection of C over AD be C', then C' lies on AB. Also, $DB = DC = DC'$, so $\angle DBC' = \angle DC'B$. Now note that

$$\angle DC'B = 180° - \angle DC'A = 180° - \angle DCA$$

which completes our proof.



Claim 1: E, D, C, B cyclic.
Proof: Note that

$$\angle FBE = \angle EBA \qquad \text{(E is incenter of } \triangle ABD) \qquad (6)$$
$$= \angle ECD \qquad \text{(AE is the angle bisector of } \angle BAC) \qquad (7)$$

Claim 2: A, B, P', D cyclic
Proof: Note that $P'D = P'B$, which follows from the definition of P'. Also note that $P'B = P'C$ implies P' is on the perpendicular bisector of segment BC, so $\angle P'AB = \angle P'AD$. Now using Lemma ($AB > AD$), we have A, B, P', D cyclic as required.
Claim 3: F, D, C, P' cyclic
Proof: Again, using the fact that P' is the circumcenter lying on the perpendicular bisector of BC we have

$$\angle P'DB = \angle P'BF = \angle P'CF \tag{8}$$

Hence we have P' coinciding with P as required.

Marker's Report

The placement of the problem and the scary-looking statement limited the number of attempts. Indeed, it is the most difficult and poorly done problem in the paper. Out of the very few who attempted, most did not draw an accurate diagram, nor made significant progress. Those who scored partials proved that E, D, C, B cyclic, but did not make any further advancements. The remaining contestants tried to use reverse-reconstruction, but most failed, as P is often confused with P'. The only successful attempt was an elaborate angle chase, but was not rewarded with full marks due to general sloppiness.

Problem 10. *A dangerous James is chasing Eric in a circular field of radius 100 meters. Eric runs 1 meter per second and James runs x meters per second. Given that James starts in the middle of the field and Eric is free to choose where to start, find the maximum x such that Eric can guarantee to not be caught forever assuming that both James and Eric can accelerate infinitely fast.*

Solution (by Brian Zhao):

We will first prove that James can catch Eric when $x > 1$. Suppose James's speed is $1 + \epsilon$, and they are separated by distance r, then James can run to Eric's starting position and trace Eric's trail. At time $\frac{r}{\epsilon}$, Eric would have travelled $\frac{r}{\epsilon}$ and James would have travelled $\frac{r}{\epsilon} + r$, thereby covering both the original distance and Eric's trail.

We will then prove that James cannot catch Eric when $x \leq 1$. Let's split time into a sequence of intervals of lengths t_1, t_2, t_3, ...

At the start of interval t_i, Eric divides the circle into two along the diameter that he lies on, and run perpendicular to the diameter into a half-circle that James is not in. Note that if James is on the diameter, then Eric can choose either. By the perpendicularity criterion, Eric's old coordinate is the closest point not in the half-circle to any point along his path, so James will not catch him within each period.

Let's set $t_i = \frac{c}{i}$, where c is a positive constant. It's well-known that $\sum_{i=1}^{\infty} t_i$ diverges, so James will never catch him.

Suppose r_i is Eric's distance to the center after time step t_i. Then $r_{i+1}^2 = r_i^2 + t_i^2$ by Pythagoras.

So $r_\infty = r_0^2 + \sum_{i=1}^{\infty} t_i^2$.

It is well-known that the infinite series converges to a constant, and since we can set r_0 and c to be arbitrarily small, Eric can stay within the circle.

Marker's Report

1. *The full solution for this problem is hard. Indeed, it took mathematicians 20 years to realize that their previous solution was a fakesolve. However, there are some easy partial marks up from grab.,*
2. *Most contestants realized that if James runs faster than Eric, then he can eventually catch*

up. However, simply stating this is not enough. We are looking for:,

3. *A well-defined strategy for James. E.g. run to Eric's starting position, then retrace Eric's path,*
4. *A few lines of math justifying that this can be done in finite time. E.g. $t =$ initial distance / (x - 1),*
5. *Many contestants stated that if Eric is faster than or equal to James, he can simply run away. This is not true. What if James back Eric into a corner?,*
6. *Some contestants thought that "infinite acceleration" meant Eric and James speed up during the game. This is not true. Their speed stayed constant. However, their velocity changed instantaneously when they turn.*